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centre, and OA, OB, OC for the directions of their semiaxes. And the attraction of such a shell on an external point may be simply expressed by means of the semiaxes of a confocal ellipsoid passing through the point. (See a Memoir by M. Chasles in Liouville's Journal, vol. v.) The quantities which we have called ρ and ρ' are, in fact, semiaxes of an ellipsoid described through the attracted point (that is, through C in the first case, and through B in the second) so as to be confocal to the surface of which the semiaxes are $a\cos\phi$, $b\cos\phi$, $c\cos\phi$."

A note by Professor Mac Cullagh, on the rotation of a solid body, was read.

Let a solid body be made to revolve round a fixed point O, and be afterwards free from any external forces; and through O conceive a right line OI to be drawn perpendicular to the invariable plane (the plane passing through O and the direction of the primitive impulse). If O be the centre of an ellipsoid whose semiaxes are in the directions of the principal axes belonging to that point, and of such lengths that the square of each semiaxis is equal to the corresponding moment of inertia divided by the mass of the body, the motion will take place in such a way that the point I, in which the right line OI intersects the surface of the ellipsoid, will be fixed in space; and therefore OI will describe within the body a cone of the second order, condirective with the ellipsoid (that is, having its circular sections parallel to those of the ellipsoid), while the point I describes on the surface of the ellipsoid a certain spherical conic. In a former number of the Proceedings (vol. ii. p. 542), the author had alluded to a theorem for determining the time at which the point I occupies any given position on the spherical conic, and he now gave a particular statement of it as follows:

Conceiving a plane of circular section of the ellipsoid to be drawn through its mean axis, and the spherical conic to be projected on this plane, first by right lines parallel to the greatest axis of the ellipsoid, and next by right lines parallel to its least axis, each projection of the conic will be a circle having its centre at O. The two projections P and Q of the same point I will always be in a right line perpendicular to the mean axis; let this right line cut the mean axis in M. Then, while the point I describes the spherical conic, the points P and Q will move in their respective circles, in such a way, that the velocity of P will vary as the ordinate MQ, and the velocity of Q will vary as the ordinate MP.

From this theorem we immediately obtain the elliptic integral which represents the time. For, supposing the greater circle to be that described by Q, and taking its radius for unity, if we put c for the radius of the other circle, and denote by ϕ the complement of the angle which OP, in any position of P, makes with the mean axis of the ellipsoid, we have $MQ = \sqrt{1 - c^2 \sin^2 \phi}$, and therefore

$$dt = \frac{k d\phi}{\sqrt{1 - c^2 \sin^2 \phi}};$$

where dt is the element of the time, and k is a constant quantity. Hence, the time at which the point P attains any given position in its circle, and therefore the time at which the point I attains any given position in the spherical conic which it describes, is determined by an elliptic function of the first kind, the modulus and amplitude of which are exhibited geometrically. The modulus c of the function is obviously the ratio of the two moduli of the cone which the right line OI describes within the body; for the radius of each circle is found by dividing the distance OI by one of the moduli of the cone.

The preceding method of determining the time in the problem of rotation occurred to the author in the year 1831, and has since been given at his lectures in Trinity College.

It may be observed, that the motion of the axis of rotation within the body is known when that of the right line OI is known; for that axis is always perpendicular to the plane which touches the ellipsoid in the point I.